MULTIMEDIA UNIVERSITY®

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210 & TERM ID 2215)

PPS0034 – INTRODUCTION TO PROBABILITY AND STATISTICS

(Foundation in Business)

15 FEBRUARY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 4 pages with **FOUR** questions.
2. Attempt **ALL** four questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided. All necessary workings **MUST** be shown.
4. **Formulae** are provided at the back of the question paper.
5. **Statistical table** is provided.

QUESTION 1

a) The discrete random variable X can take only the values 1, 3 and 5. For these values, the **cumulative distribution function, $F(X)$** is defined by

$$F(X) = \begin{cases} \frac{3x^2+5m}{100} & x = 1,3,5 \end{cases}$$

i) Find the value of m. (2 marks)

ii) Construct the **probability distribution of X, $P(X)$.** (2 marks)

iii) Find the expected value and variance for the random variable X. (6 marks)

b) The continuous random variable Y has the following probability density function:

$$f(y) = \begin{cases} \frac{1}{3}ky & ; \ 0 \leq y < 3 \\ k(4-y) & ; \ 3 \leq y \leq 4 \\ 0 & ; \ \text{otherwise} \end{cases}$$

i) Show that $k = \frac{1}{2}$. (5 marks)

ii) Find the mean and variance for the random variable Y. (10 marks)

(Total 25 marks)

QUESTION 2

a) The probability that Elsa wakes up before her alarm rings is 0.58.

i) Find the mean and variance of the number of times that Elsa wakes up before her alarm rings, in the next 9 mornings. (3 marks)

ii) Determine the probability that in the next 10 mornings, Elsa will wake up before her alarm rings in more than 1 but less than 5 mornings. (4 marks)

iii) Calculate the probability that in the next 2 weeks Elsa will **NOT** wake up before her alarm rings on at most 8 mornings. (2 marks)

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b) Defects occur at random in planks of wood with a constant rate of 0.6 per 10cm length. Jean buys a plank of length 100cm.

i) Find the probability that Jean's plank contains at least 3 defects. (3 marks)

Sharon buys 8 planks each of length 100cm.

ii) Find the probability that from 3 to 7 of Sharon's planks contain at least 3 defects. (5 marks)

c) Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test.

i) Tom takes the test and scores 585. Will he be admitted to this university? (4 marks)

ii) What is the probability that Tom's score will be between 378 and 628? (4 marks)

(Total 25 marks)

QUESTION 3

a) Consider the following data on the prices for the hard cover dictionaries available on the website for a major seller.

RM89.90 RM129.90 RM109.90 RM129.90 RM109.90 RM89.90

i) List all the possible samples of size five (without replacement) from this population and construct the sampling distribution of the sample mean. Then, find the sampling error for each sample. (15 marks)

ii) If a random sample of 5 prices: 89.90, 129.90, 129.90, 109.90, 89.90 was mistakenly recorded as 89.90, 129.90, 119.90, 109.90, 89.90, calculate the non-sampling error. (3 marks)

b) Suppose the mean length of time that a caller is placed on hold when telephoning a customer service center is 23.8 seconds, with standard deviation 4.6 seconds. Find the probability that the mean length of time on hold in a sample of 1,200 calls will be within 0.5 second of the population mean. (7 marks)

(Total 25 marks)

Continued...

QUESTION 4

a) The average number of days to complete recovery from a particular type of knee operation is 134.5 days. From his experience, a physician claims that the use of a topical medication might be shortening the recovery time. He randomly selects the records of forty-eight knee surgery patients who used the topical medication. The times to total recovery were:

151	126	121	128	128	135	142	123
135	122	127	150	129	143	149	137
134	133	131	132	125	124	141	146
148	144	130	133	151	129	134	139
139	136	117	118	120	119	125	128
122	148	120	138	140	121	119	123

Assume that the population variance is 97.8120 days.

Assuming a normal distribution of recovery times, does the data support the physician's claim at the 0.2% level of significance.

(10 marks)

b) The board of a major internet service company requires that the mean wait time for customers for service calls is not more than 3.08 minutes. To make sure that the mean wait time is not exceeding the requirement, an assignment manager tracks the wait times of 52 randomly selected calls. The mean wait time was calculated to be 3.64 minutes. Assuming the population standard deviation is 1.42 minutes, is there sufficient evidence to support the requirement at the 0.04 significance level?

(9 marks)

c) A parent advocacy group is concerned about the amount of time children spend each week doing school homework at home. A simple random sample of 58 students had a mean of 10.7 hours per week doing homework at home after school. Construct a 92% confidence interval for the mean number of hours per week a student spends doing homework at home. Assume that the population standard deviation is 1.76 hours per week.

(6 marks)

(Total 25 marks)

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Formulae:

1.

	Mean	Variance
Discrete Random Variable X	$\mu = E(X)$ $= \sum xP(x)$	$Var(X) = E(X^2) - [E(X)]^2$ **where** $E(X^2) = \sum x^2 P(x)$
Continuous Random Variable X	$\mu = E(X)$ $= \int_{-\infty}^{\infty} xf(x)dx$	$Var(X) = E(X^2) - [E(X)]^2$ **where** $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

2.

	Formula	Mean	Standard Deviation
Binomial Probability	$P(x) = \binom{n}{x} p^x q^{n-x}$	$\mu = np$	$\sigma = \sqrt{npq}$
Poisson Probability	$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$	$\mu = \lambda$	$\sigma = \sqrt{\lambda}$

3. The z value for a value of x: $z = \frac{x - \mu}{\sigma}$

4. The z value for a value of $\bar{x}$: $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$

   where $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

5. Sampling error = $\bar{x} - \mu$
   Non-sampling error = incorrect $\bar{x}$ − correct $\bar{x}$

6. Point estimate of $\mu = \bar{x}$

   Margin of error = $\pm 1.96\sigma_{\bar{x}} = \pm 1.96\frac{\sigma}{\sqrt{n}}$ or $= \pm 1.96 s_{\bar{x}} = \pm 1.96\frac{s}{\sqrt{n}}$

7. The $(1-\alpha)100\%$ confidence interval for μ is

   $\bar{x} \pm z\sigma_{\bar{x}}$ if σ is known

   $\bar{x} \pm z s_{\bar{x}}$ if σ is not known

   where $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ & $s_{\bar{x}} = \frac{s}{\sqrt{n}}$

End of Paper